

*Reference: F&H chapter 18*

### Introduction:

- Over half the known nuclei have configurations **(Z,N) even,  $J^\pi = 0^+$**
- Recall that an empirical **pairing term** is included in the semi-empirical mass formula to account for their **unusual stability**.
  - N.B. **The pairing term is *not* accounted for in the shell model, which ignores all interactions between particles!!!**
- **It costs too much energy to break a pair** of nucleons and populate higher single particle states, so the excitations of even-even nuclei tend to be of a **collective nature**
  - **the nuclear matter distribution as a whole exhibits quantized vibrations in some cases and rotations in others, with characteristic frequency patterns.**
- **Vibrational spectra** are seen in nuclei that have an intrinsic **spherical shape**
- **Rotational excitations** tend to occur in nuclei with permanent **quadrupole deformations**

**Model:** quantized oscillations of a liquid droplet at constant density  
(why? repulsive short-distance behaviour of the N-N force!)

Consider oscillations about a spherical equilibrium shape, with a time-dependent boundary surface expressed as a linear combination of spherical harmonic functions:

$$R(\theta, \phi, t) = R_o \left\{ 1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}(\theta, \phi) \right\}$$

expansion describes any shape at all, given appropriate coefficients. Each contribution can in principle oscillate at a different frequency....

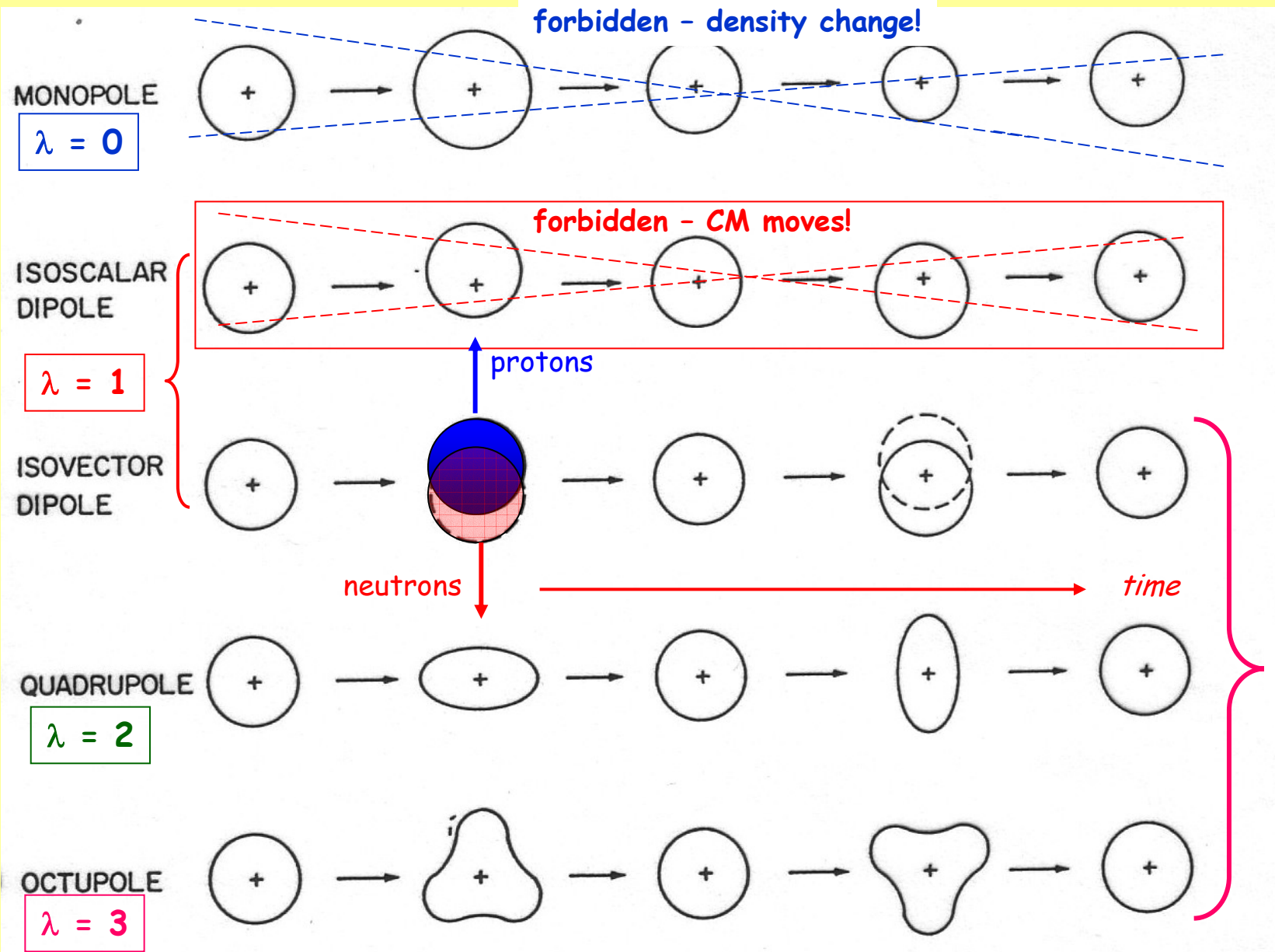
Normal modes of the system correspond to excitations with a particular value of  $\lambda$  and  $\mu$ , and these will occur at characteristic frequencies.

Application to nuclei:

1. restriction to axial symmetry, i.e.  $\mu = 0$
2. vibrations are quantized:  $E_n = n \hbar \omega$

# Illustration: time sequence of oscillating nuclear shapes

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- characteristic feature that can be seen in **all nuclei**
- very short-lived state at **high excitation**
- $E_1 = \hbar\omega_1 \cong 78 A^{-1/3} \text{ MeV}$  (example below:  $^{197}\text{Au}$ ,  $E = 15 \text{ MeV}$ )
- $\Gamma \cong 6 \text{ MeV}$  (common feature)  $\rightarrow \tau \cong 10^{-22} \text{ s}$
- classical analog is an **oscillating electric dipole moment** - can decay via E1 radiation pattern (signature)

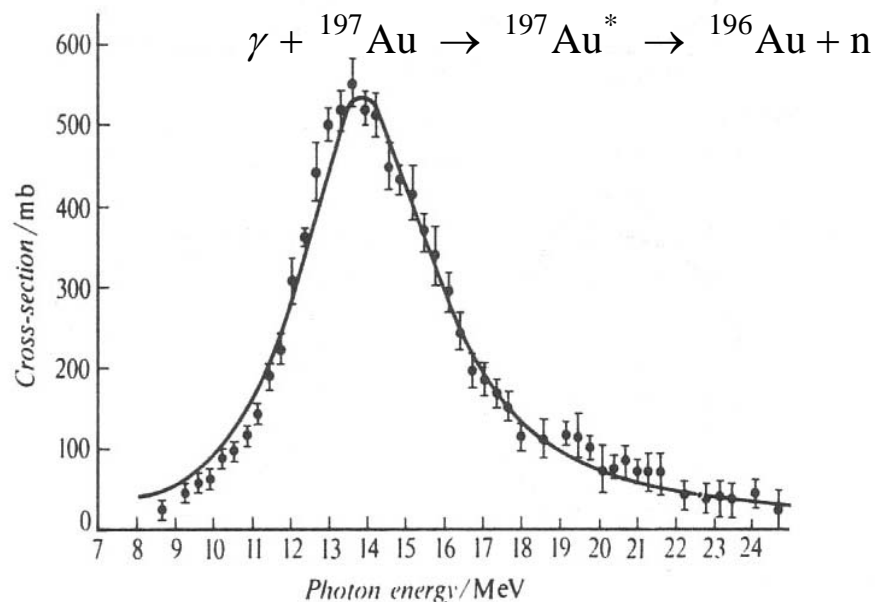


Fig. 8.10 Giant resonance of photodisintegration in  $^{197}\text{Au}$ . The yield of neutrons is shown as a function of the energy of the monochromatic photons used to produce the reaction (Fultz, S. C. et al., *Phys. Rev.*, **127**, 1273, 1963).

Electric dipole radiation pattern:

*Griffiths, Intro. to Electrodynamics:*

$$\langle \vec{S} \rangle \sim \frac{\sin^2 \theta}{r^2} \hat{r}$$

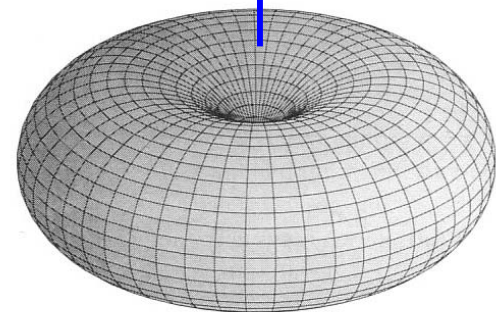
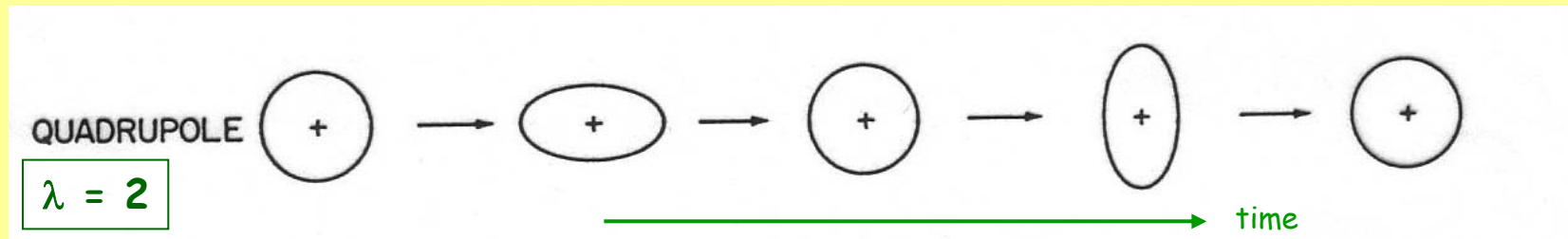


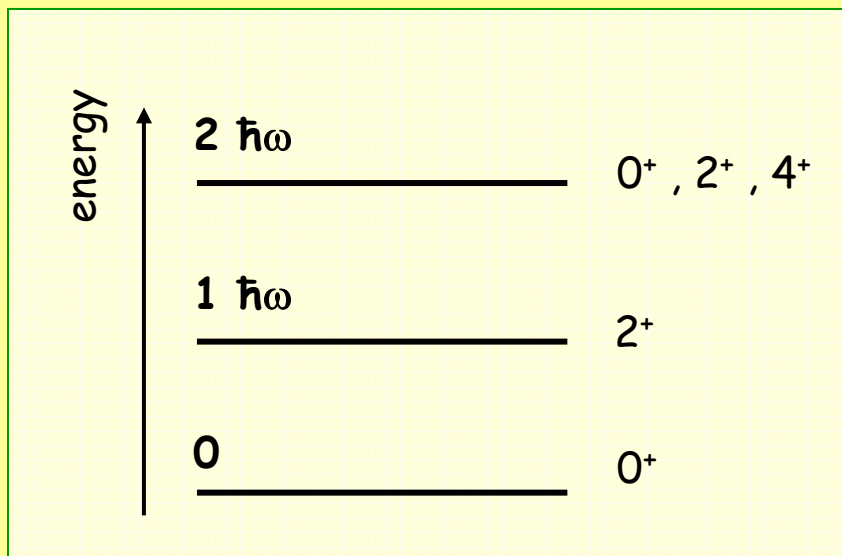
Figure 11.4

Quadrupole oscillations occur at lower energy:  $J^\pi = 2^+$

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- typically,  $\hbar\omega_2 \sim 1$  MeV in a variety of even-even nuclei
- excitation energy is low, so can expect to see up to several “quadrupole phonons” in the spectrum
- **Boson** excitations, so require a **symmetric wave function** under exchange of “particle” (phonon) labels  $\rightarrow$  this **restricts the total  $J^\pi$** ,



e.g. for two phonons:

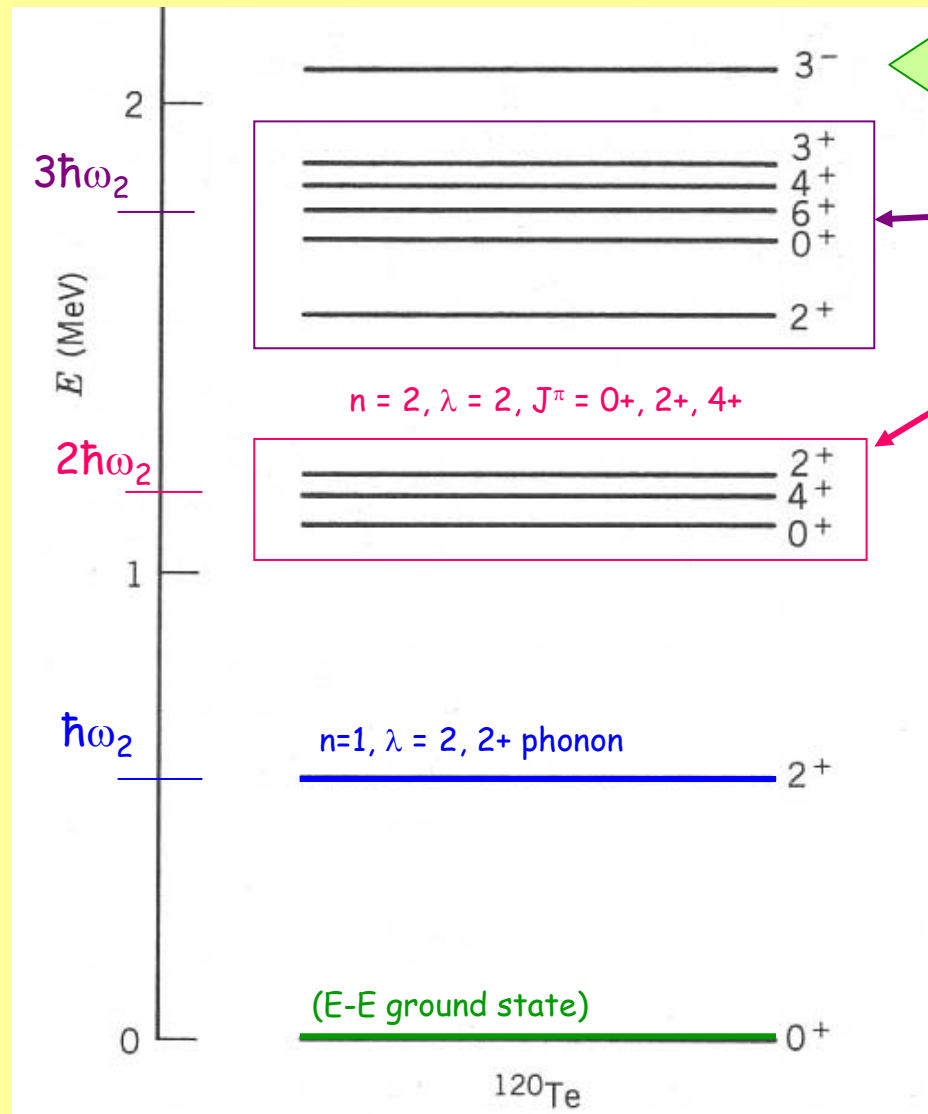
$$\left( \vec{2} + \vec{2} \right)_{\text{symmetric}} = \vec{J}$$
$$\Rightarrow J = 0, 2, 4 \text{ only}$$

Model spectrum

# Example of vibrational excitations:

$^{120}_{52}\text{Te}_{68}$

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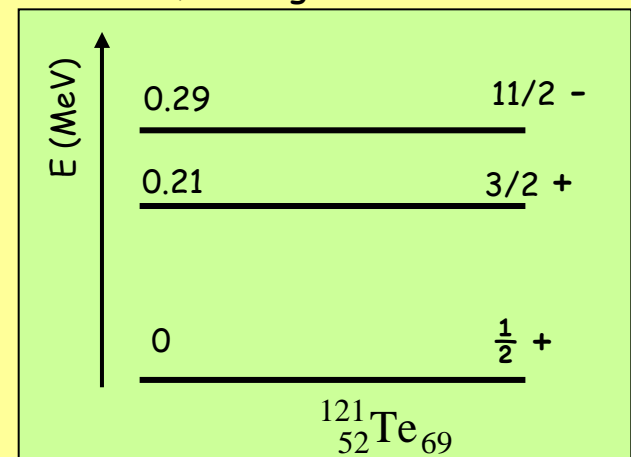


3- state?

multiple  $\lambda = 2$  phonon states, ideally degenerate

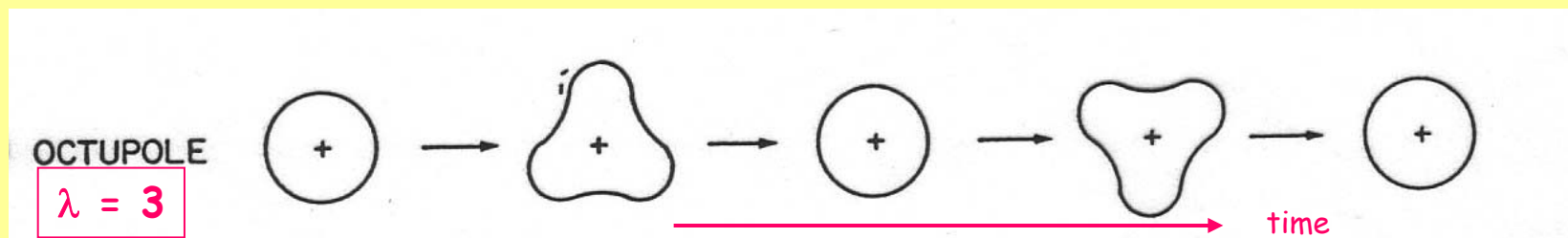
(characteristic decay patterns: gamma-rays emitted with "E2" angular distributions, like a quadrupole antenna)

contrast, adding one neutron:



The  $3^-$  state is an octupole phonon,  $\lambda = 3$ :

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- $J^\pi = 3^-$
- $\hbar\omega_3 \sim (2-3) \hbar\omega_2 \sim 2 - 3 \text{ MeV}$
- typically only see one octupole phonon per spectrum

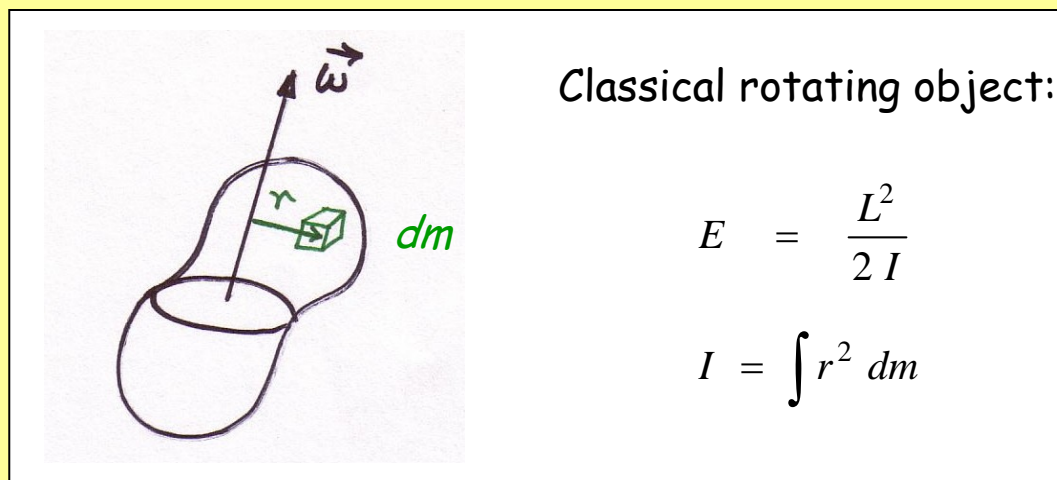
### Summary:

- low lying excitations in even-even spherical nuclei have the same characteristic pattern up to a few MeV in excitation energy:

$0^+$  (gs),  $2^+$  (quadrupole phonon),  $(0^+, 2^+, 4^+)$  (two phonons),  $3^-$  (octupole)

## 2. Quantized Rotations in deformed nuclei:

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Replace  $L$  with rotational angular momentum  $J$ :

$$|J^2| = J(J+1) \Rightarrow E_J = \frac{\hbar^2}{2I} J(J+1)$$

moment of inertia  
sets the scale of the  
energy level pattern

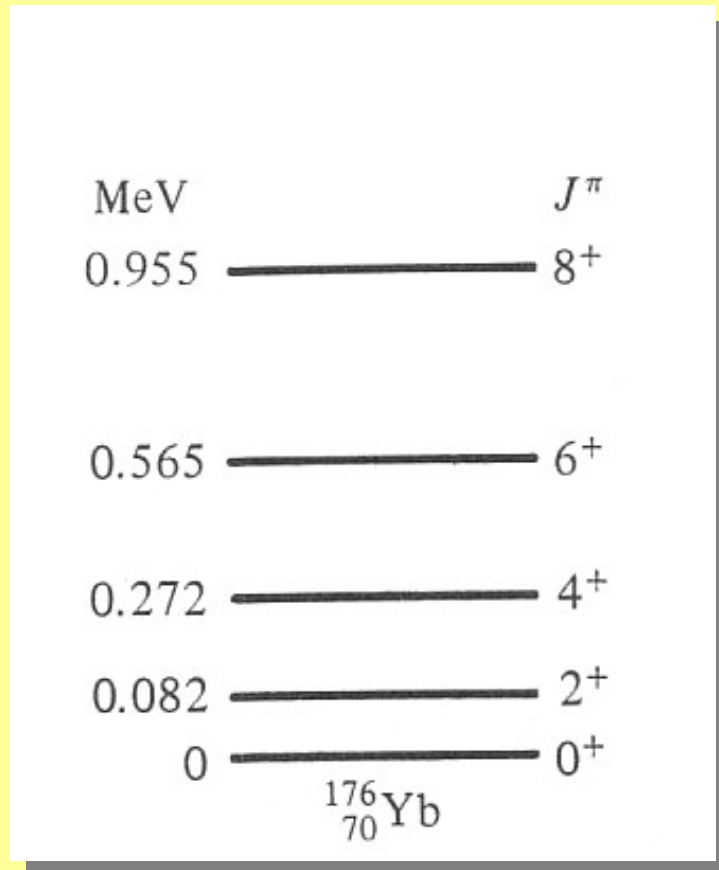
allowed  $J$  determine  
a characteristic  
spacing pattern

$J$  is quantized; "rotational bands" are spectra characterized by a given value of the moment of inertia,  $I$ , and a series of energy levels with  $\Delta J = 1$  or  $2$ :

- even-even nucleus:  $J = (0, 2, 4, 6, 8, 10, \dots)$   $\pi = +$
- odd-even deformed nucleus:  $J = \frac{1}{2}$  integer,  $\Delta J = 1$  within a "band"



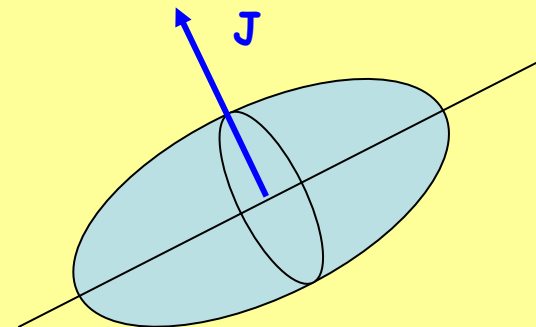
## Quantized energy states of a rotating football!



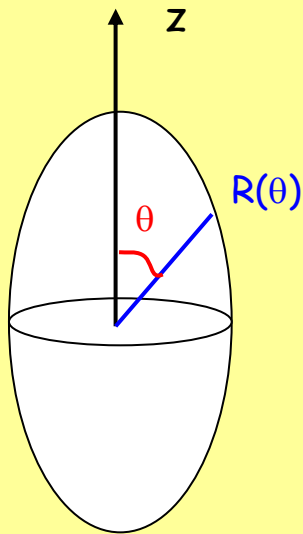
Note -- larger  $I$  means smaller energy level spacing!

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

$$\Rightarrow \frac{\hbar^2}{2I} \cong 0.014 \text{ MeV}$$



note: rotations around the symmetry axis are indistinguishable; rotational angular momentum must be **perpendicular** to the symmetry axis.



parameterize the shape, quadrupole moment and moment of inertia assuming constant density football shape:

$$R(\theta) = R_o (1 + \beta Y_{20}(\theta))$$

$$\beta = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_o} \cong 1.05 \frac{\Delta R}{R_o}$$

$$Q_{zz} = \frac{3}{\sqrt{5\pi}} R_o^2 Z\beta (1 + 0.16\beta)$$

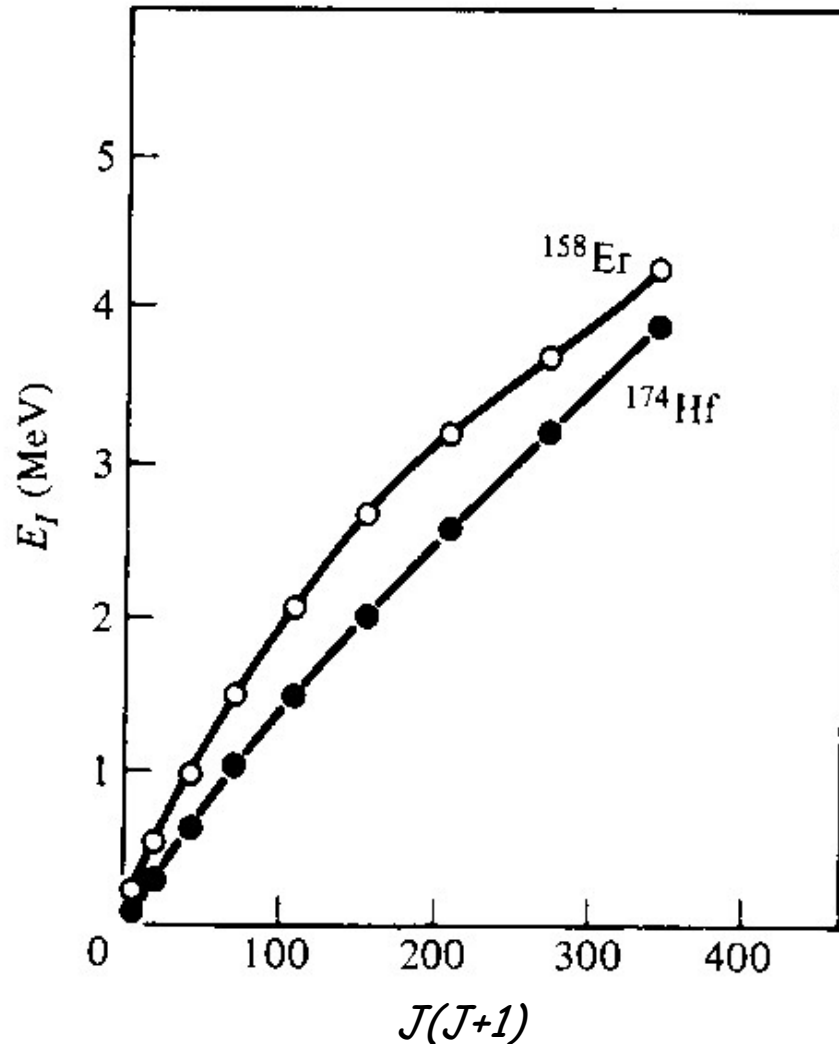
If the nucleus rotates like a solid: (*rigid body model*):

$$I_R = \frac{2}{5} M R_o^2 (1 + 0.31\beta)$$

If the nucleus rotates like a liquid drop: (*rotating fluid model*)

$$I_F = \frac{9}{8\pi} M R_o^2 \beta$$

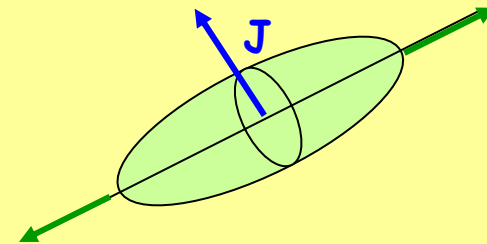
reality is  
somewhere  
in between...



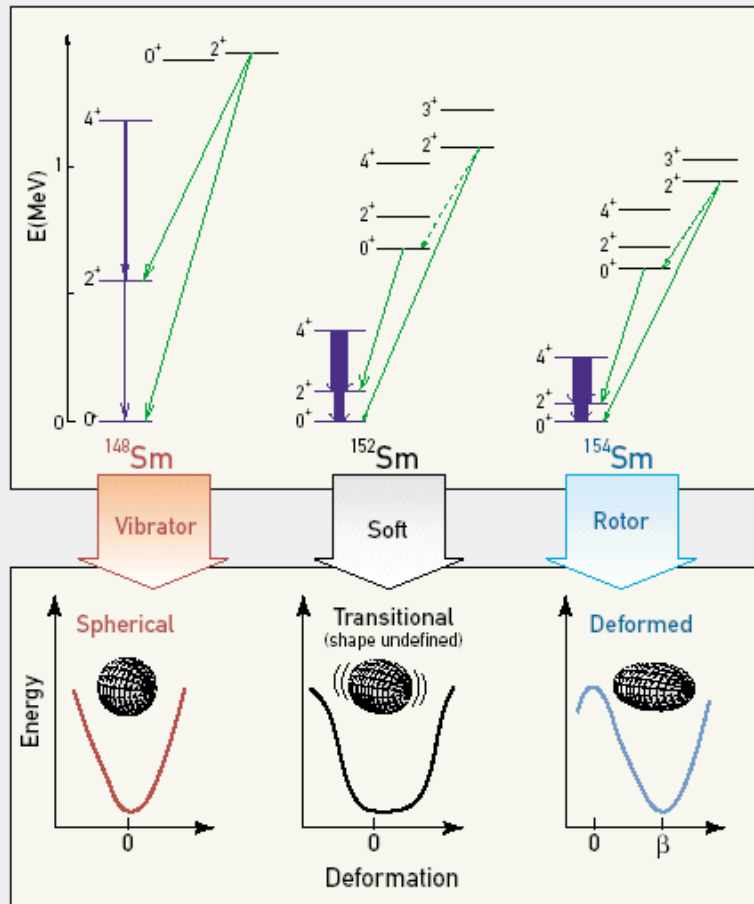
spectral analysis:

a plot of  $E$  vs  $J(J+1)$  should give a straight line with slope  $\hbar^2/2I$

- confirmed for  $^{174}\text{Hf}$
- but for  $^{158}\text{Er}$ , the slope decreases (moment of inertia increases) with increasing  $J$  ...

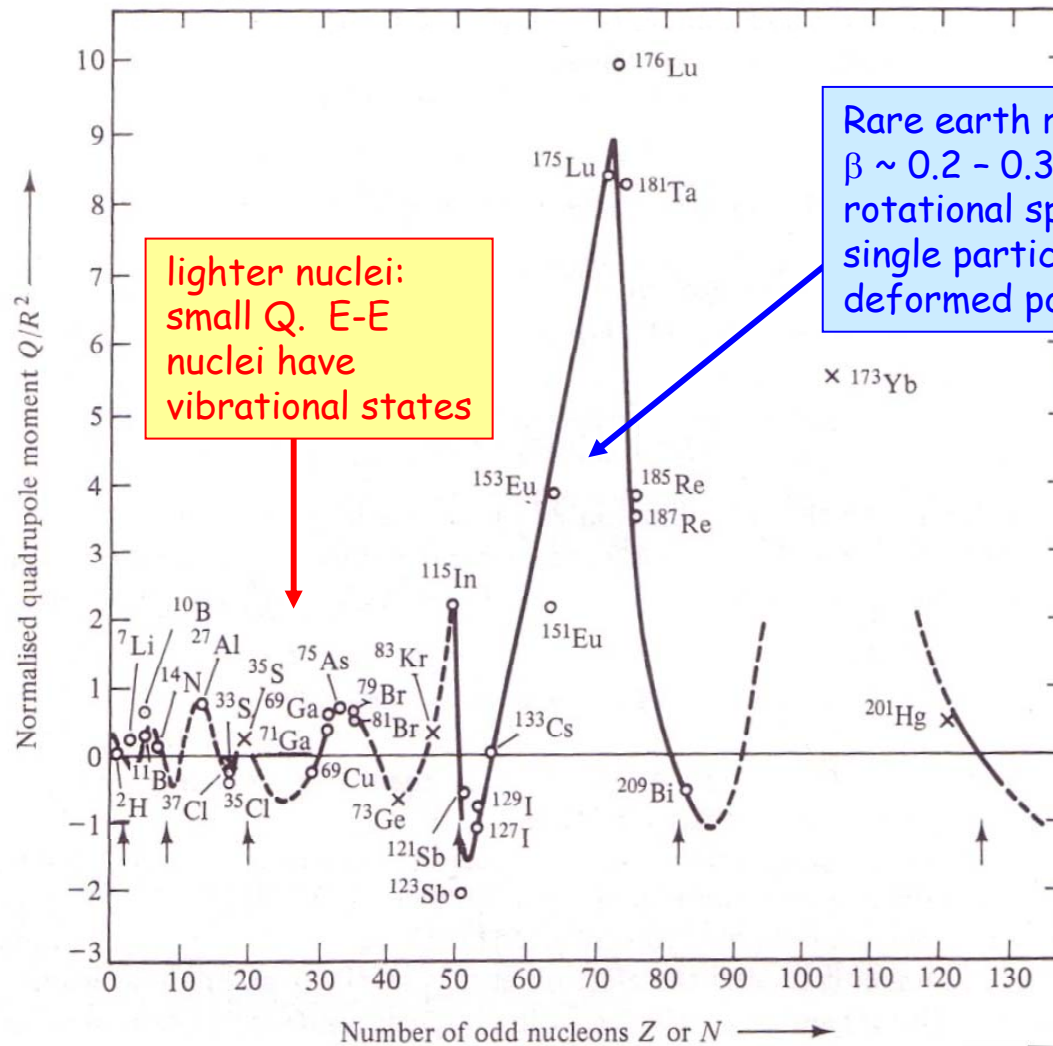


Like a rotating fluid:  
“centrifugal stretching” along  
the symmetry axis occurs for  
larger angular momentum!



[web link: see lecture 1](#)

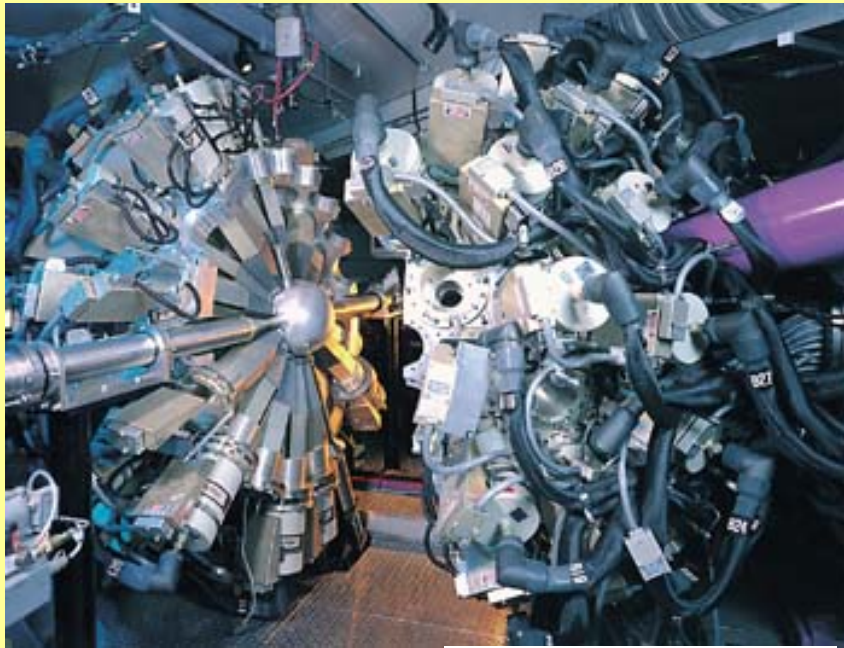
**Shapes of samarium.** The energy-level schemes for three excited isotopes of samarium are shown at the top of the figure, and the inferred nuclear shapes are shown below. The isotope  $^{148}\text{Sm}$  shows features characteristic of a spherical vibrator, whereas  $^{154}\text{Sm}$  exhibits rotational bands typical of a deformed (elongated) nucleus. On the other hand,  $^{152}\text{Sm}$  behaves like a critical-point system, whose shape cannot be precisely defined. This is illustrated in the energy diagrams below the shapes, where energy is plotted as a function of shape deformation. Well-defined minima exist for two of the isotopes, but the energy minimum for  $^{152}\text{Sm}$  is very broad, and it is impossible to say whether the nucleus is spherical or deformed.



lighter nuclei:  
small  $Q$ . E-E  
nuclei have  
vibrational states

Rare earth region:  
 $\beta \sim 0.2 - 0.3$ ,  
rotational spectra for E-E  
single particle states in a  
deformed potential for O-E

Closed shell nuclei:  
spherical; O-E nuclei  
near closed shells are  
good "Shell Model"  
candidates...



Gammasphere Detector

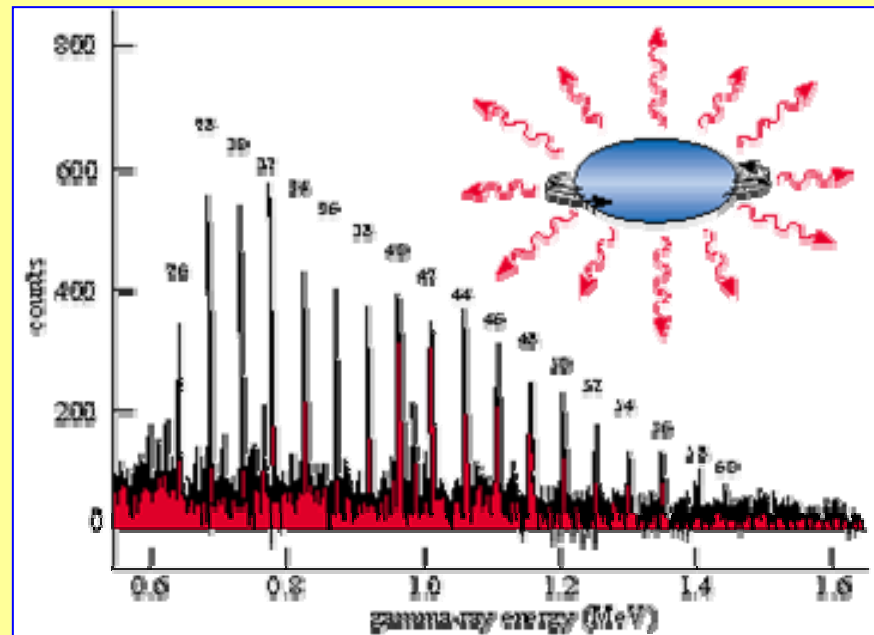
e.g. "8  $\pi$ " detector array at TRIUMF,  
"EUROBALL" array in France, "Gammasphere" UK

- high resolution detectors: Ge crystals
- surrounded by low resolution, high efficiency scintillation detectors (typically NaI) to reject events where only part of the energy is deposited in the Ge crystal due to Compton scattering
- full angular coverage for both  $\rightarrow$  "8  $\pi$  detection!"

## Great current interest: "Superdeformed Nuclei" (2:1 axis ratio)

Physics World, July 1998: Superdeformed nuclei of dysprosium-152 decay by emitting a regular spectrum of gamma-rays. The number above each transition is the angular momentum quantum number, which decreases by two each time a photon is emitted. The photon carries  $h/p$  of angular momentum away from the nucleus, which slows down the rotation. After emitting approximately 20 such gamma-rays the nucleus abruptly loses its deformation.

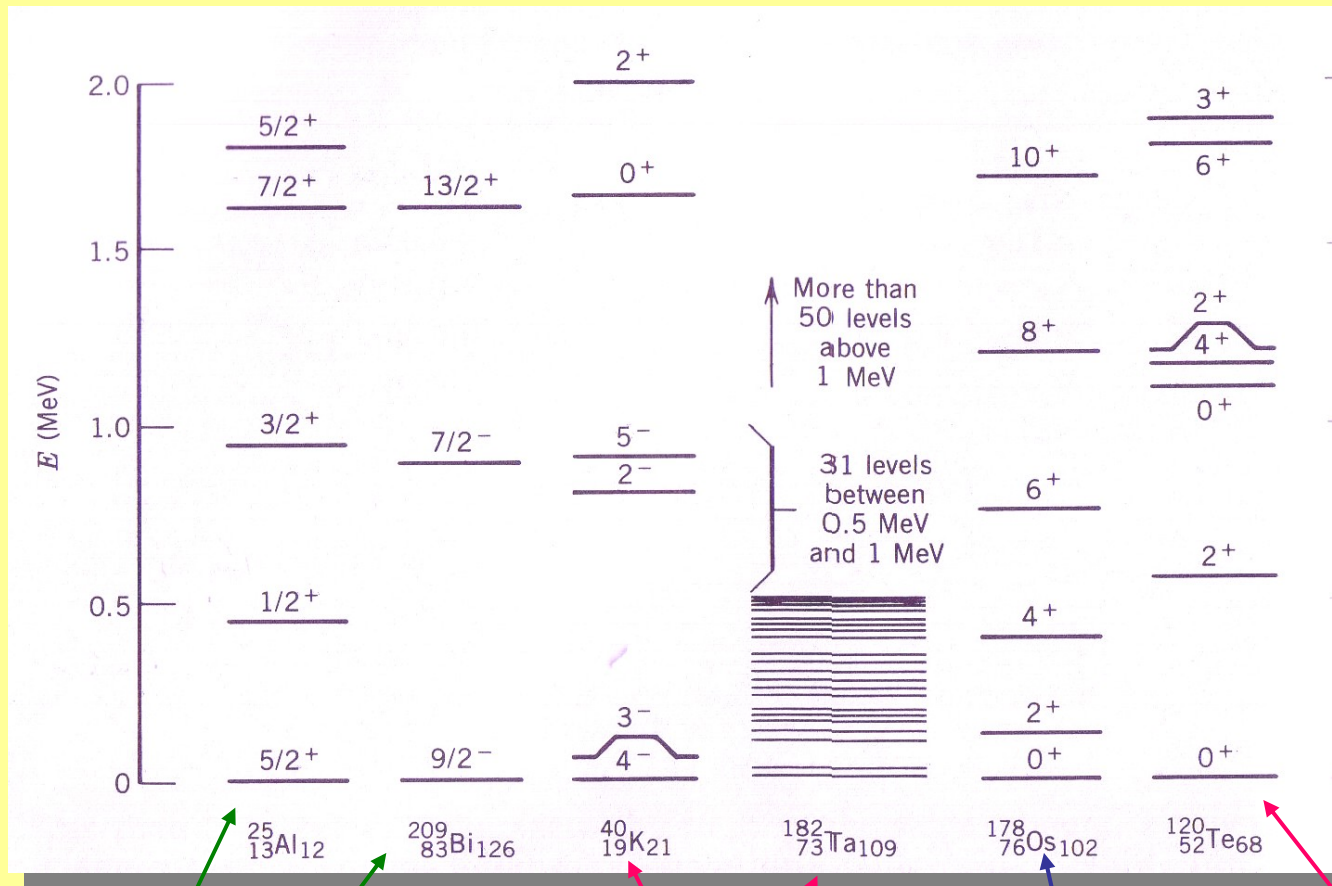
Data from CLRC Daresbury Laboratory, UK





## Example - nuclear states:

15



light nuclei, 1 valence nucleon  
- good shell model description

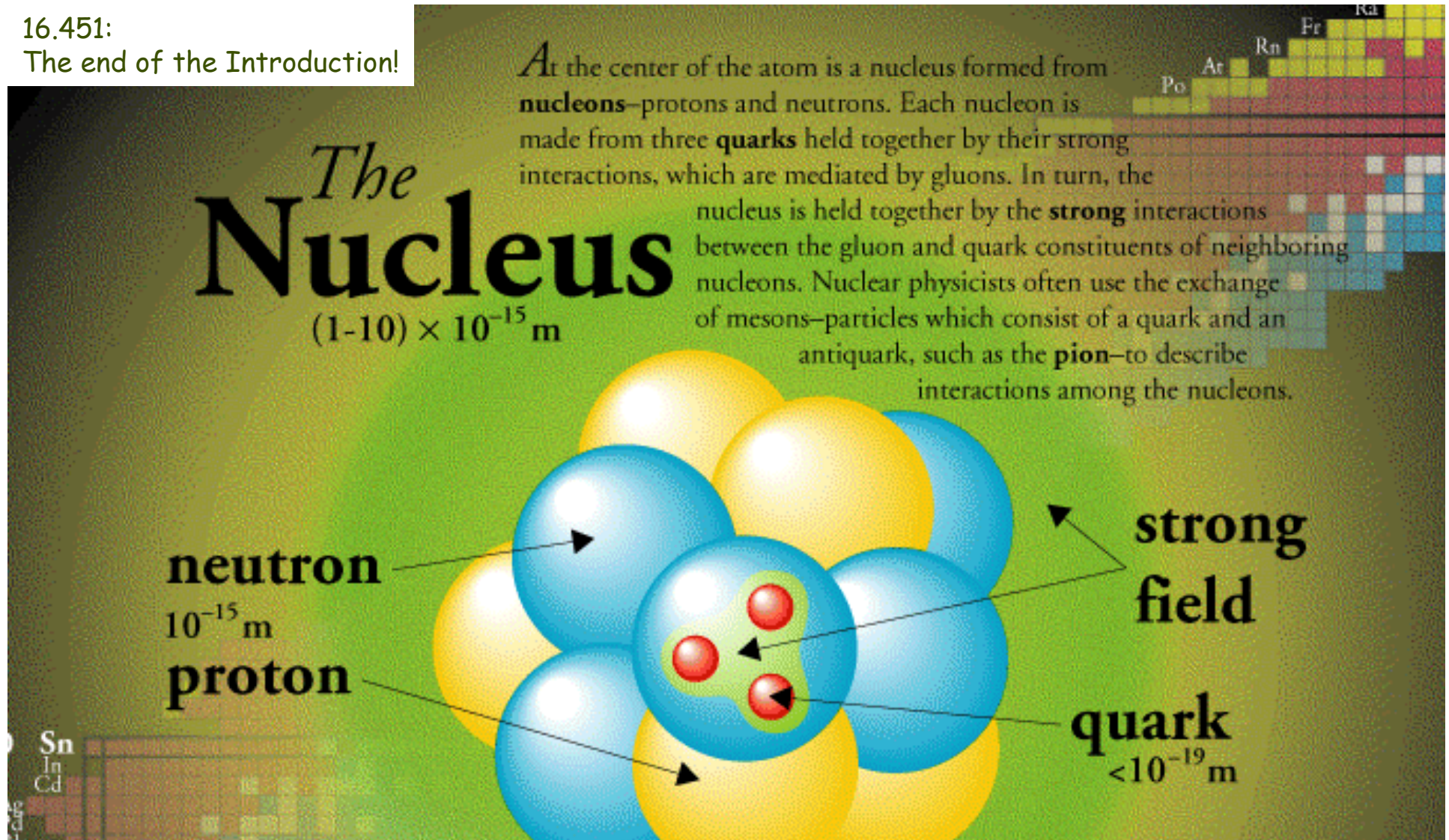
odd-odd - more than  
one valence nucleon  
-- complicated!!!

rotational  
spectrum

vibrational  
spectrum

16.451:

The end of the Introduction!



From the U.S. Long range plan : “Opportunities in Nuclear Science” (lecture 1):

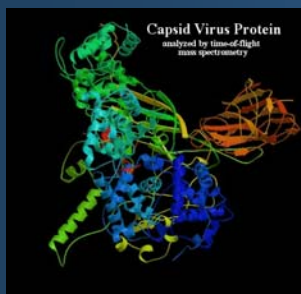
The nucleus is a remarkable quantal system displaying diverse and rare phenomena. Governed by the strong interactions among nucleons, nuclei exhibit correlations resulting in both single-particle and collective modes of excitation. Nuclear structure theory attempts to understand these excitations and the responses of nuclei to different external probes, within a coherent framework. This theoretical framework must encompass a wide range of energy and momentum scales for nuclei ranging from the deuteron to the superheavy elements. Theory strives to describe the structure and dynamics of these often-disparate systems and to apply the understanding thus achieved to unravel some of the mysteries of the universe....





## Graduate Studies & Research

- Astronomy & Astrophysics
- Atomic & Molecular Physics
- Condensed Matter & Materials Physics
- Mass Spectrometry of Biomolecules
- Medical Physics
- Physics of Nanoscale Systems
- Subatomic Physics



*CURRENT PROJECTS IN SUBATOMIC PHYSICS:*  
*theory of nucleon structure;*  
*symmetry tests of strong & weak interactions;*  
*nuclear astrophysics,*  
*precise atomic mass determinations;*  
*ion & atom trapping ....*

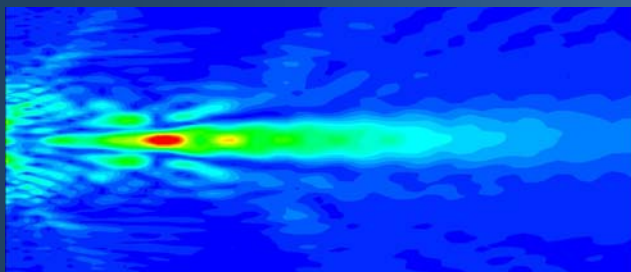
### STRATEGIC LINKS:

*TRIUMF, Argonne & Los Alamos National Labs,*  
*Jefferson Lab, Max Planck Inst. ....*

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